Statistics

Spring 2023

Lecture 23

Feb 19-8:47 AM

Introductions to counting:

Select one number from 0 to 9.

$$\underbrace{0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9}_{10 \text{ choices}}$$

Now let's select two numbers with replacement

10 choices

00	01	02	03	- - - -	09	
10	11	12	13	- - - -	19	
20	21	22	23	- - - -	29	10 choices
⋮						
90	91	92	93	- - - -	99	

First Selection · Second Selection

$$\underbrace{10 \quad \cdot \quad 10}_{100 \text{ Selections}}$$

P(I guess correctly what numbers you selected)

$$= \frac{1}{100}$$

Now select 2 numbers $\boxed{\text{without replacement}}$

First Selection · Second Selection

$$\underbrace{10 \quad \cdot \quad 9}_{90 \text{ choices in total.}}$$

Mar 20-7:18 AM

How many ways can we select a passcode of 4 digits?

1) Repetition allowed

First · Second · Third · 4th

$$\underbrace{10 \cdot 10 \cdot 10 \cdot 10}$$

$10^4 = 10{,}000$ choices.

2) No repetition

First · Second · Third · Fourth

$10 \cdot 9 \cdot 8 \cdot 7$

$= 5040$ choices

Mar 20-7:24 AM

think of passcode with a letter followed by 3 digits.

1) Not case sensitive, and repetition allowed.

letter · digit · digit · digit

$26 \cdot 10 \cdot 10 \cdot 10$

$= 26000$ choices

2) Case sensitive and no repetition allowed

A a

B b

⋮

Z z

letter · digit · digit · digit

$52 \cdot 10 \cdot 9 \cdot 8 = \boxed{33280}$ choices

Mar 20-7:28 AM

Consider 5 Students
Adam, Bill, Carol, David, Eddie
Select 2 of them

~~AB~~	~~AC~~	~~AD~~	~~AE~~
BA	~~BC~~	~~BD~~	~~BE~~
CA	CB	~~CD~~	~~CE~~
DA	DB	DC	~~DE~~
EA	EB	EC	ED

Total 20 Selections

If order does not matter

10 Choices ✓

If we have n different items, and we need r of them, and order does not matter,

$$\#\text{ of Selections} \quad {}_nC_r = \frac{n!}{r! \cdot (n-r)!}$$

Last example
5 Students
2 " to be Selected
and order does not matter

$${}_5C_2 = \frac{5!}{2! \cdot (5-2)!} = \frac{5!}{2! \cdot 3!} = \frac{5 \cdot 4 \cdot \cancel{3 \cdot 2 \cdot 1}}{2 \cdot 1 \cdot \cancel{3 \cdot 2 \cdot 1}} = \boxed{10}$$

Mar 20-7:33 AM

Find ${}_5C_2$ using TI:

5 [MATH] [→] PRB [↓] [${}_nC_r$] 2 [Enter]

$\boxed{10}$

Suspose we have [10 different items] (n), and we like to Select [4 items] (r) order does not matter, NO replacement,
How many ways can this be done?

${}_{10}C_4$

10 [MATH] [→] PRB [↓] [${}_nC_r$] 4 [Enter]

$\boxed{210}$

Select 5 numbers, no repetition, order does not matter from 1 to 50.

$n = 50$
$r = 5$

\# Selections ${}_{50}C_5 = \boxed{2{,}118{,}760}$

Select 8 numbers, no repetition, order does not matter from 1 to 100. Scien. Notat.

$n = 100$
$r = 8$

\# Selections ${}_{100}C_8 = \boxed{1.86 \times 10^{11}}$

Mar 20-7:42 AM

A deck of playing cards has 52 Cards and 12 face Cards.

1) How many ways can we draw 3 Cards, No replacement?

$$_{52}C_3 = \boxed{22100}$$

2) How many was can we draw 3 face Cards, no replacement?

$$_{12}C_3 = \boxed{220}$$

3) $P(\text{Select 3 face Cards}) = \dfrac{\text{Total face Selections}}{\text{Total Selections}}$

$$= \frac{_{12}C_3}{_{52}C_3} = \frac{220}{22100} = \boxed{\frac{11}{1105}}$$

$\approx \boxed{.010}$

Rare event

$0 < P(\text{rare event}) \leq .05$

Mar 20-7:52 AM

A piggy bank has 15 nickels and 5 dimes. $n = 20$

randomly take 2 coins, No replacement. $r = 2$

1) How many ways can this be done?

$$_{20}C_2 = 190$$

2) How many ways can we select 2 dimes?

$$_5C_2 = 10$$

3) P(Selecting 2 dimes, order does not matter, No replacement)

$$= \frac{_5C_2}{_{20}C_2} = \frac{10}{190} = \boxed{\frac{1}{19}} \approx \boxed{.053}$$

Mar 20-7:59 AM

4 Females, 11 Males $n=15$

we need to Select 3 people $r=3$ order does not matter.

1) How many ways can this be done?

$_{15}C_3 = 455$

2) How many ways can we select 1 Female & 2 Males?

FMM
MFM
MMF

$_4C_1 \cdot {}_{11}C_2 = 220$

3) $P(\text{Selecting } 1F \,\&\, 2M) = \frac{_4C_1 \cdot {}_{11}C_2}{_{15}C_3} = \frac{220}{455} = \boxed{\frac{44}{91}}$

4) $P(\text{Selecting } 2F \,\&\, 1M) = \frac{_4C_2 \cdot {}_{11}C_1}{_{15}C_3} = \boxed{\frac{66}{455}}$

$\approx \boxed{.145}$

Mar 20-8:04 AM

Consider a $\underbrace{\text{Standard deck of playing cards}}_{\text{52 Cards, 26 Red, 12 Face, 4 Aces}}$

Draw 5 Cards, order does not matter, no replacement.

$P(2 \text{ Aces } \& \text{ } 3 \text{ Face}) = \frac{_4C_2 \cdot {}_{12}C_3}{_{52}C_5} = \frac{132\cancel{0}}{259896\cancel{0}}$

$\frac{132}{259896} = \frac{66}{129948} = \frac{33}{64974} = \boxed{\frac{11}{21658}} \approx 5.1 \times 10^{-4}$

Divide by 2 divide by 2 Divide by 3

$P(3 \text{ Aces } \& \text{ } 2 \text{ Face}) = \frac{_4C_3 \cdot {}_{12}C_2}{_{52}C_5} = \frac{264}{2598960}$

$= \boxed{1.02 \times 10^{-4}}$

Mar 20-8:12 AM